OPTIMAL SPATIAL SEARCHES WITH LONG-RANGE TUNNELING

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ABSTRACT

A quantum walk on a lattice is a paradigm of a quantum search in a database. The database qubit strings are the lattice sites, qubit rotations are tunneling events, and the target site is tagged by an energy shift [1–3]. For continuous-time quantum walks, the walker diffuses across the lattice and the search ends when it localizes at the target site. The search time T can exhibit Grover's optimal scaling [4,5] with the lattice size N, namely, $T \sim \sqrt{N}$, on an all-connected, complete lattice. For finite-range tunneling between sites, instead, Grover's optimal scaling is warranted when the lattice is a hypercube, as illustrated in Fig. 1(a), of dimension $d > 4$ [3]. In this symposium, we will show that Grover's optimum can be reached in lower dimensions on lattices of long-range interacting particles, when the interaction strength scales algebraically with the distance r as $1/r^\alpha$ and $0 < \alpha < 3d/2$, see Fig. 1(b) [6]. For $\alpha < d$ the dynamics mimics the one of a globally connected graph. For $d < \alpha < d+2$, the quantum search on the graph can be mapped to a short-range model on a hypercube with spatial dimension $d_s = 2d/(\alpha - d)$, indicating that the search is optimal for $d_s > 4$. The critical spectral dimension $d_s = 4$ corresponds to exponent $\alpha = \alpha_c = 3d/2$, which is associated with a phase transition in the time complexity of the search problem. The corresponding order parameter χ_α, with its squared value being the fidelity, is shown in Fig. 2. Our work [6] hereby identifies an exact relation between criticality of long-range and short-range systems, it provides a quantitative demonstration of the resources that long-range interactions provide for quantum technologies, and indicates when existing experimental platforms can implement efficient analog quantum search algorithms.

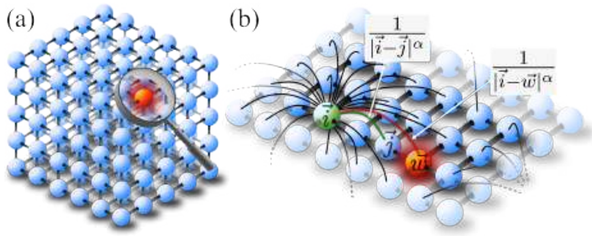

Figure 1: (a) Illustrative graphic of search on a cubic lattice (hypercube with $d = 3$) with nearest-neighbor couplings ($\alpha \to \infty$). Target node for which we search is depicted in red. (b) Schematic of the power-law scaling of the connectivity of a single site $\vec{i}$ in a two-dimensional cubic lattice.

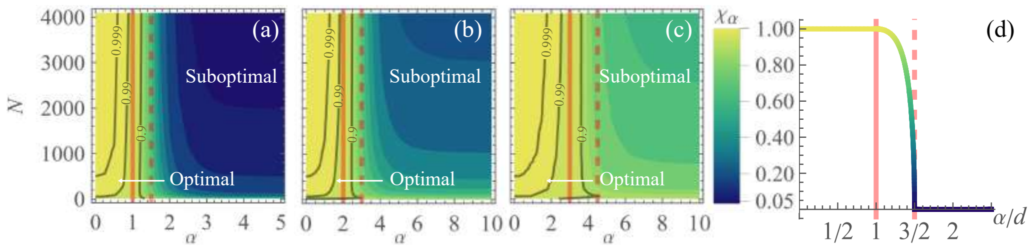

Figure 2: *Upper bound to the search fidelity:* χ_α as a function of the number of lattice sites N and long-range tunneling exponent α for (a) $d = 1$, (b) $d = 2$, and (c) $d = 3$. Vertical lines correspond to $\alpha = d$ (solid) and $\alpha = \alpha_c = 3d/2$ (dashed). (d) Asymptotic behavior of χ_α in the limit $N \to \infty$ as a function of α/d. For $0 < \alpha < d$, $\chi_\alpha = 1$, while for $d < \alpha < 3d/2$ it decreases monotonically to zero as $\chi_\alpha = \sqrt{3 - 2\alpha/d}/(2 - \alpha/d)$.

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